پاسخ لرزه‌ای سیستم SDOF با رفتار خطی :

6.2 معادله دیفرانسیل حاکم بر مسئله :

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

$$\ddot{u} + 2\xi\omega_n \dot{u} + \omega_n^2 u = -\ddot{u}_g(t) \qquad \omega_n^2 = \frac{k}{m} \; ; \; \frac{c}{2m} = \xi\omega_n$$

پاسخ سیستم در زمان t تنها تابع ω_n (یا T_n) و ξ است.

$$u = u(t, \xi, \omega_n)$$

برای حل معادله دیفرانسیل تنها حل عددی مفید است و روش های فصل 5 مورد استفاده قرار می‌گیرد. راه حل تحلیلی وجود ندارد.

6.3. کمیت های پاسخ : مهم ترین پاسخ ها $u(t)$: تعیین نیروهای داخلی اعضا، که از روی آن نیروی برشی و لنگر خمشی حاصل می‌شود

برای ماشین آلات و دستگاه های حساس به جابجایی، شتاب : $u^t(t) = u_g(t) + u(t)$; $\ddot{u}^t$

6.4 تاریخچه پاسخ ها : 1) با افزایش پریود سازه، زمان لازم برای طی یک مرحله کامل افزایش می‌یابد. این زمان نزدیک به پریود طبیعی سازه است (اگرچه فرکانس های تحریک لرزه‌ای متعدد هستند)

2) با افزایش پریود جابجایی افزایش می‌یابد البته این نتیجه برای همه پریودها قطعی و صادق نیست.

3) در پریود ثابت، با افزایش نسبت میرایی دامنه جابجایی کاهش می‌یابد.

زمانی که تاریخچه پاسخ مشخص شده باشد ارزیابی نیروهای داخلی به سادگی امکان پذیر است:

نیروی معادل استاتیکی $f_s(t) = k\,u(t) = m\omega_n^2 u(t) = m\,A(t)$

$A(t) = \omega_n^2 u(t)$

شبه شتاب — نیروی معادل استاتیکی

برای یک قاب یک طبقه :

نیروهای داخلی

$$\begin{cases} V_b(t) = f_s(t) = m\,A(t) \\ M_b(t) = h\,f_s(t) = h\,V_b(t) \end{cases}$$

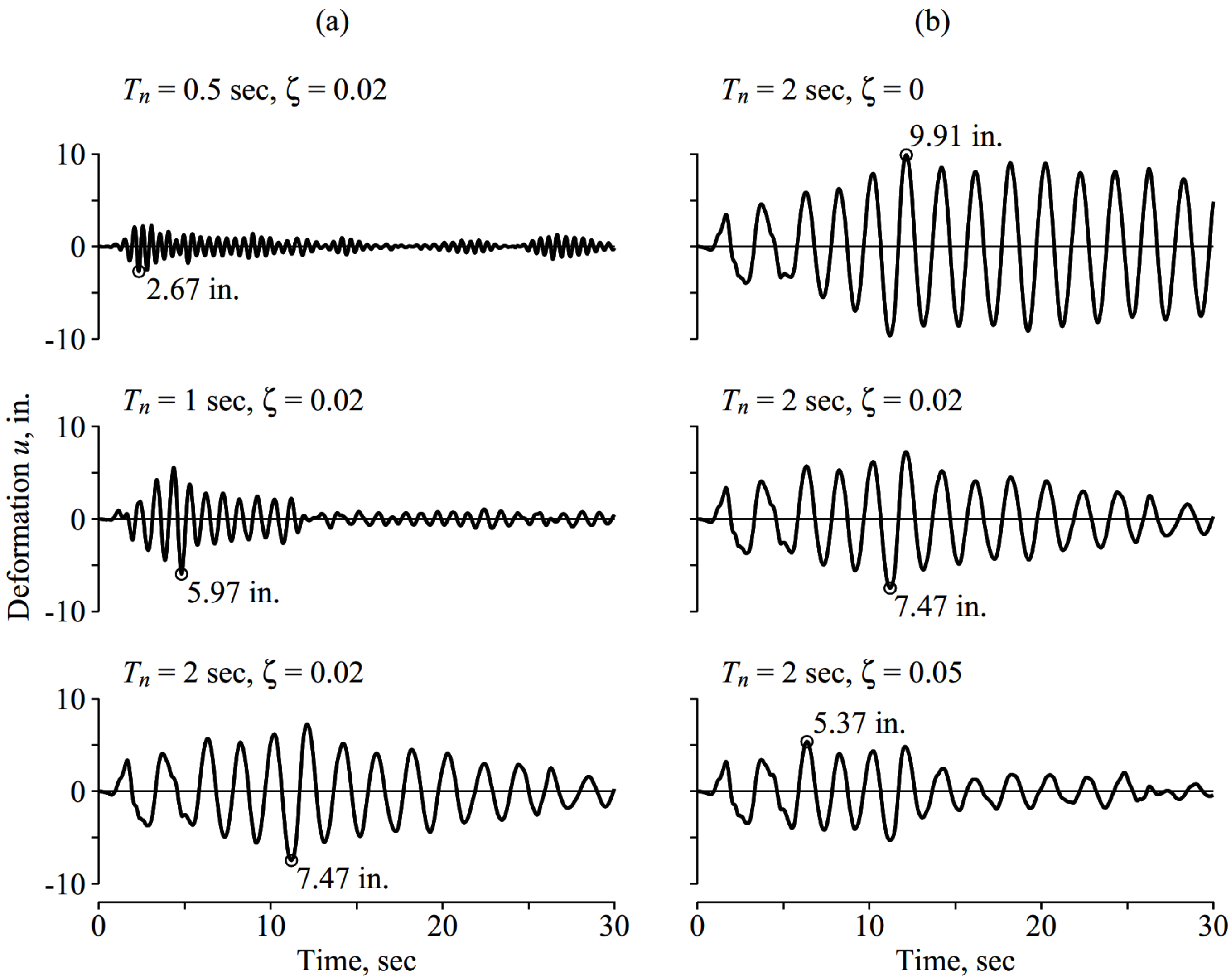


Figure 6.4.1 Deformation response of SDF systems to El Centro ground motion.

$$f_S(t) = ku(t) \tag{6.4.1}$$

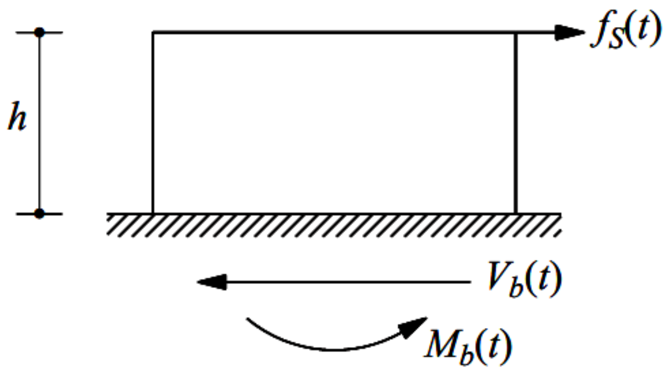


Figure 6.4.2 Equivalent static force.

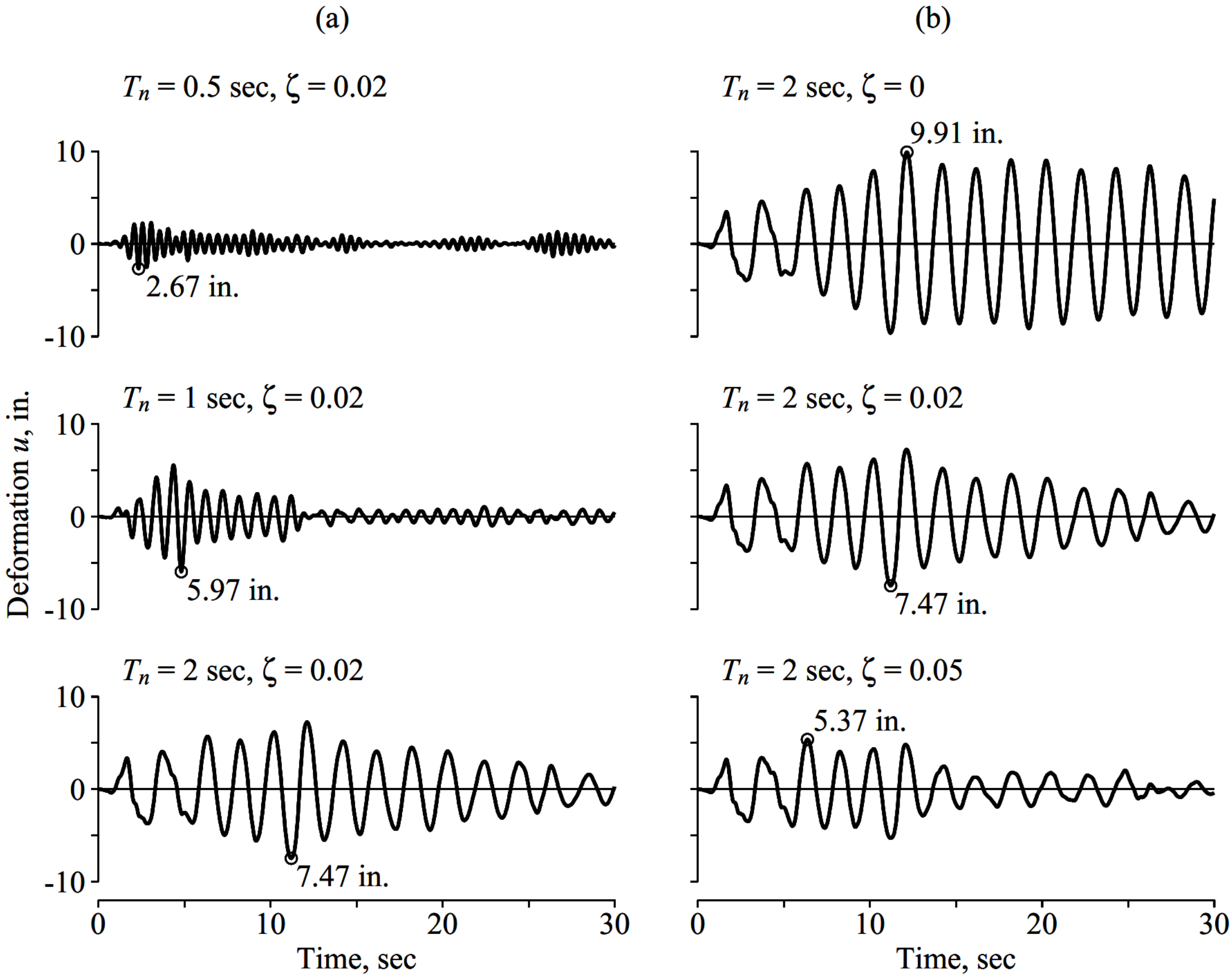


Figure 6.4.1 Deformation response of SDF systems to El Centro ground motion.

6.5 مفهوم طیف پاسخ: ترسیم نمودار تغییرات یک کمیت پاسخ برحسب پریود یا فرکانس طبیعی سیستم در نسبت میرایی مشخص طیف پاسخ نامیده می شود.

$$u_o(T_n,\xi_n) = \max_t |u(t,T_n,\xi_n)|$$

$$\dot{u}_o(T_n,\xi_n) = \max_t |\dot{u}(t,T_n,\xi_n)|$$

$$\ddot{u}^t_o(T_n,\xi_n) = \max_t |\ddot{u}^t(t,T_n,\xi_n)|$$

6.6 جابجایی، شبه سرعت و شبه شتاب:

☑ شبه شتاب متناسب با نیروی معادل استاتیکی است

$D \equiv u_o \qquad V = \omega_n D$

☑ شبه سرعت متناسب با انرژی ذخیره شده کرنشی است.

$$E_{so} = \frac{1}{2} k u_o^2 = \frac{1}{2} m \omega_n^2 u_o^2 = \frac{1}{2} m V^2$$

از جنس انرژی پتانسیل — از جنس انرژی جنبشی

☑ استفاده از لفظ شبه شتاب برای ایجاد تفاوت بین A و $\ddot{u}_o$ است.

$$A = D\omega_n^2 = D\left(\frac{2\pi}{T_n}\right)^2$$

$$V_{bo} = f_{so} = k u_o = m\omega_n^2 D = mA$$

نیروی حاصل از سختی — نیروی اینرسی

$$V_{bo} = \frac{A}{g} w$$

A/g دربرگیرنده و معرف ضریب برش پایه در آیین نامه هاست.

به این ترتیب طیف پاسخ نمایش تغییرات D، V، A برحسب T_n در نسبت میرایی مشخص ξ است.

طیف ترکیبی پاسخ D-V-A :

هر سه طیف مذکور دربرگیرنده اطلاعات مشابه با نمایش متفاوت هستند و با در دست داشتن یکی، دو مورد دیگر به راحتی با عملیات ریاضی حاصل می شوند. اما هر یک تفسیر فیزیکی متفاوتی بدست می دهند:

D: متناسب با جابجایی

V: متناسب با انرژی کرنشی ذخیره شده $\quad V = D\omega_n$

A: متناسب با نیروی معادل استاتیکی $\quad A = V\omega_n = D\omega_n^2$

نمایش طیف به صورت سه جانبه اطلاعات کاملی از وضعیت و پارامترهای طیف بدست می دهد.

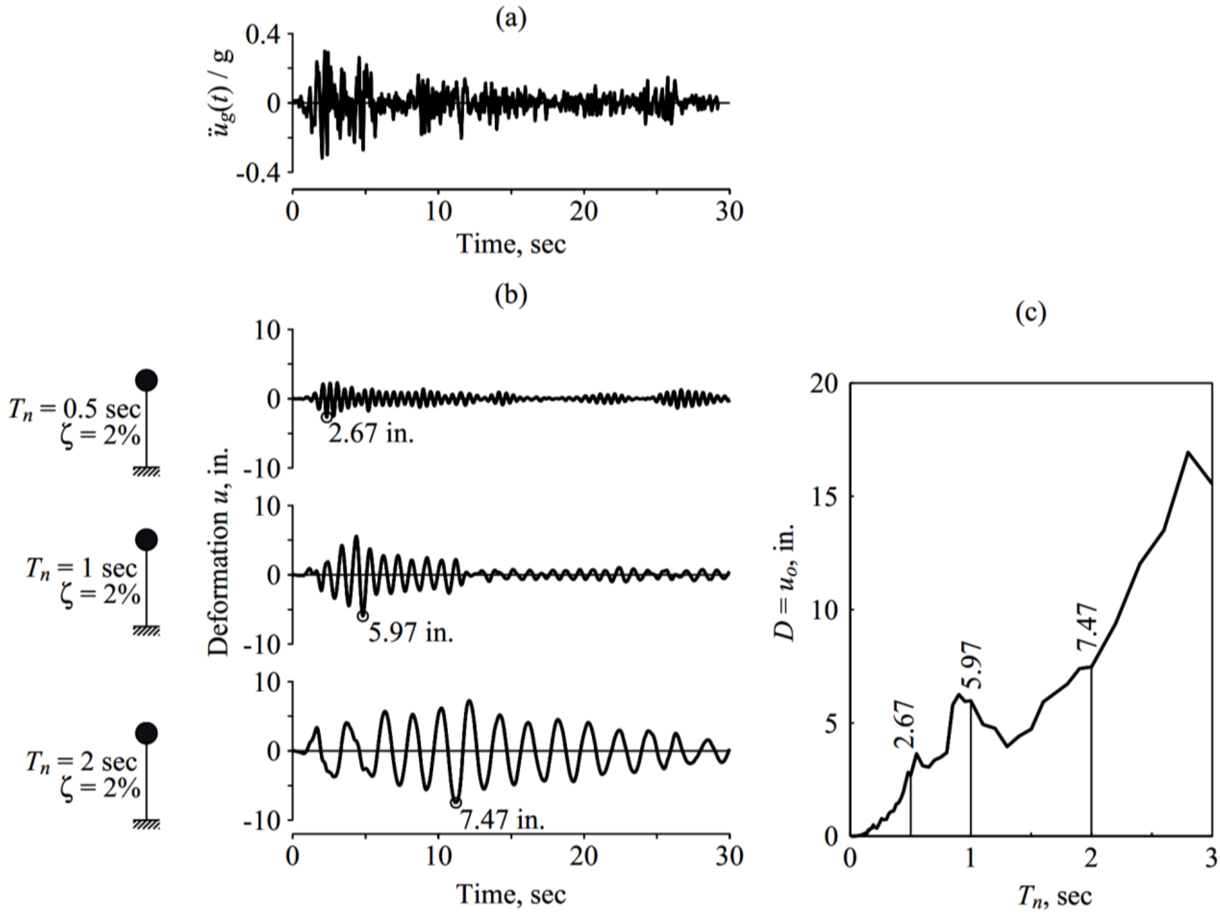


Figure 6.6.1 (a) Ground acceleration; (b) deformation response of three SDF systems with $\zeta = 2\%$ and $T_n = 0.5$, 1, and 2 sec; (c) deformation response spectrum for $\zeta = 2\%$.

later, the complete response spectrum includes such spectrum curves for several values of damping.

6.6.2 Pseudo-velocity Response Spectrum

Consider a quantity V for an SDF system with natural frequency ω_n related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$V = \omega_n D = \frac{2\pi}{T_n} D \tag{6.6.1}$$

The quantity V has units of velocity. It is related to the peak value of strain energy E_{So} stored in the system during the earthquake by the equation

$$E_{So} = \frac{mV^2}{2} \tag{6.6.2}$$

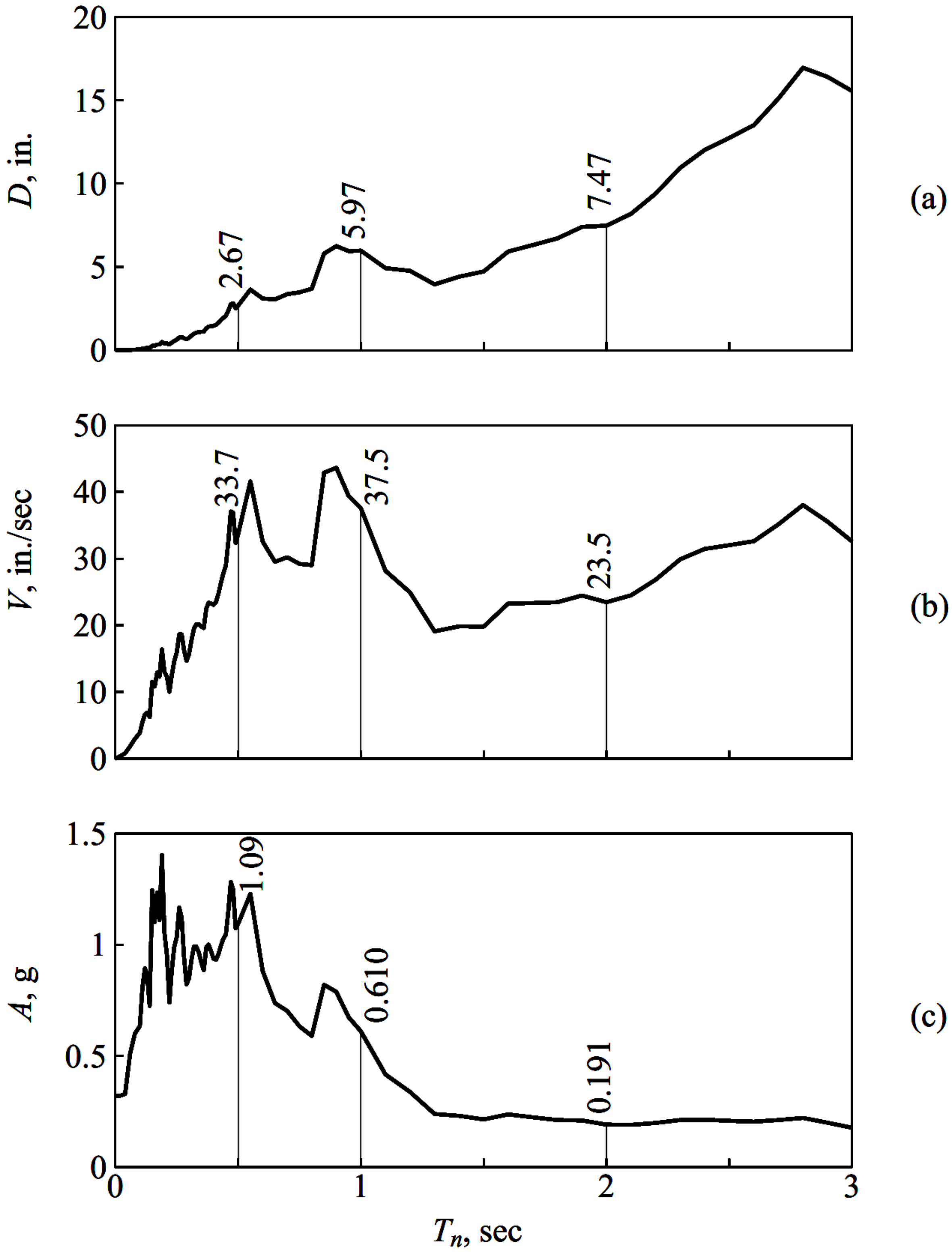


Figure 6.6.2 Response spectra ($\zeta = 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectum; (c) pseudo-acceleration response spectrum.

6.6.4 Combined *D–V–A* Spectrum

Each of the deformation, pseudo-velocity, and pseudo-acceleration response spectra for a given ground motion contains the same information, no more and no less. The three spectra are simply different ways of presenting the same information on structural response. Knowing one of the spectra, the other two can be obtained by algebraic operations using Eqs. (6.6.1) and (6.6.3).

Why do we need three spectra when each of them contains the same information? One of the reasons is that each spectrum directly provides a physically meaningful quantity. The deformation spectrum provides the peak deformation of a system. The pseudo-velocity spectrum is related directly to the peak strain energy stored in the system during the earthquake; see Eq. (6.6.2). The pseudo-acceleration spectrum is related directly to the peak value of the equivalent static force and base shear; see Eq. (6.6.4). The second reason lies in the fact that the shape of the spectrum can be approximated more readily for design purposes with the aid of all three spectral quantities rather than any one of them alone; see Sections 6.8 and 6.9. For this purpose a combined plot showing all three of the spectral quantities is especially useful. This type of plot was developed for earthquake response spectra, apparently for the first time, by A. S. Veletsos and N. M. Newmark in 1960.

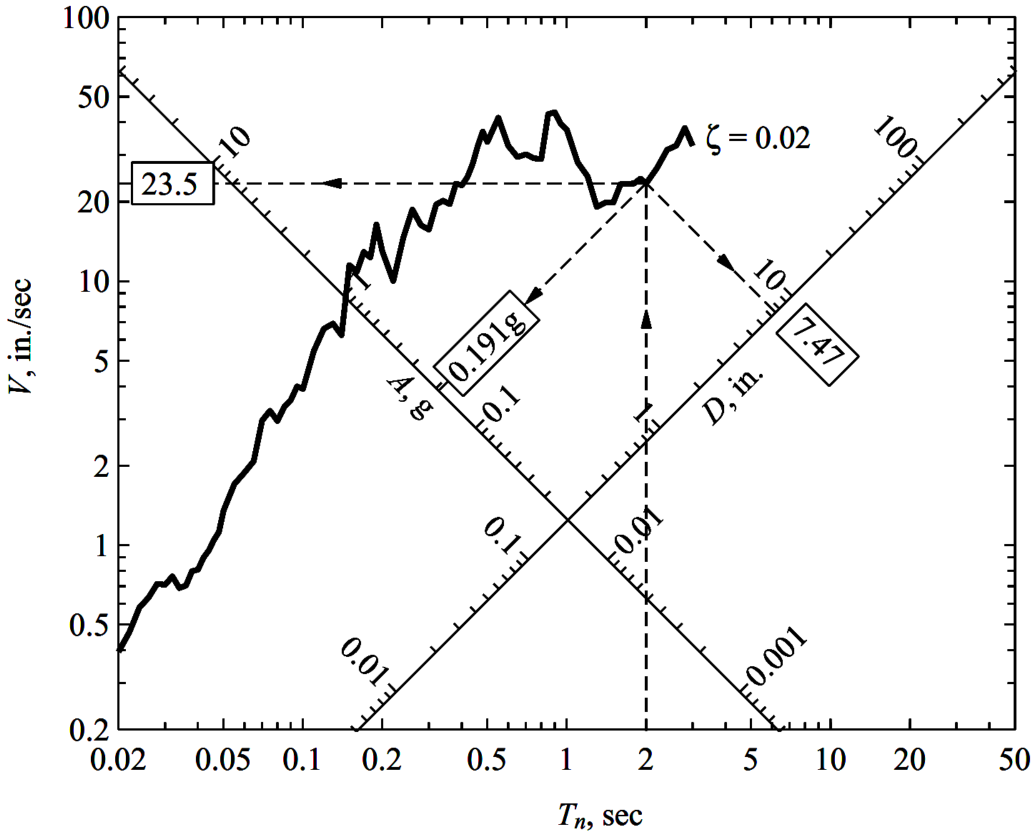


Figure 6.6.3 Combined *D–V–A* response spectrum for El Centro ground motion; $\zeta = 2\%$.

This integrated presentation is possible because the three spectral quantities are interrelated by Eqs. (6.6.1) and (6.6.3), rewritten as

$$\frac{A}{\omega_n} = V = \omega_n D \quad \text{or} \quad \frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D \tag{6.6.6}$$

Observe the similarity between these equations relating D, V, and A and Eq. (3.2.21) for the dynamic response factors R_d, R_v, and R_a for an SDF system subjected to harmonic excitation. Equation (3.2.21) permitted presentation of R_d, R_v, and R_a, all together, on four-way logarithmic paper (Fig. 3.2.8), constructed by the procedure described in Appendix 3 (Chapter 3). Similarly, the graph paper shown in Fig. A6.1 (Appendix 6) with four-way logarithmic scales can be constructed to display D, V, and A, all together. The vertical and horizontal scales for V and T_n are standard logarithmic scales. The two scales for D and A sloping at $+45°$ and $-45°$, respectively, to the T_n-axis are also logarithmic scales but not identical to the vertical scale; see Appendix 3.

Once this graph paper has been constructed, the three response spectra—deformation, pseudo-velocity, and pseudo-acceleration—of Fig. 6.6.2 can readily be combined into a single plot. The pairs of numerical data for V and T_n that were plotted in Fig. 6.6.2b on

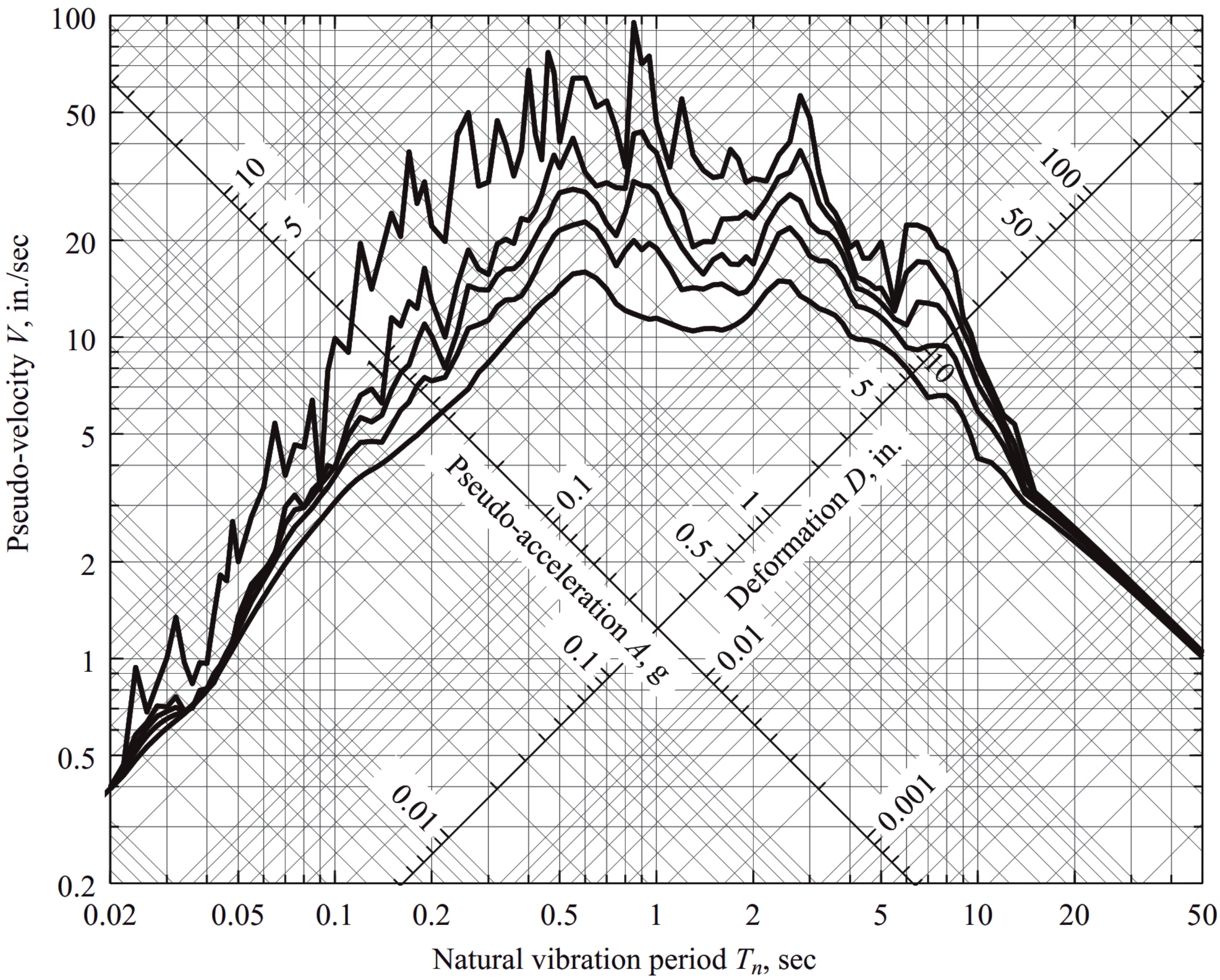


Figure 6.6.4 Combined D–V–A response spectrum for El Centro ground motion; $\zeta = 0$, 2, 5, 10, and 20%.

6.6. ساختن طیف پاسخ: فرض کنیم شتاب حرکت زمین $\ddot{u}_g(t)$ به صورت عددی مشخص شده باشد. در این صورت، گام های زیر برای ساختن طیف طی می شوند:

1. عددی ساختن $\ddot{u}_g(t)$ در گام زمانی مشخص، مثلاً $\Delta t = 0.02\,s$
2. انتخاب نسبت میرایی ζ و پریود طبیعی T_n برای یک سیستم یک درجه آزادی (SDF)
3. محاسبه تاریخچه جابجایی $u(t)$ برای سیستم SDF تحت تحریک $\ddot{u}_g(t)$ با استفاده از یک روش عددی
4. محاسبه بیشینه پاسخ جابجایی در تاریخچه زمانی، u_o $\quad u_o = \max_t |u(t)|$
5. محاسبه پارامترهای طیفی: $D = u_o \; ; \; V = \omega_n D \; ; \; A = \omega_n^2 D$
6. تکرار عملیات از گام 2 تا 5 برای نسبت های میرایی مورد نظر ζ و T_n
7. نمایش گام ها و نتایج حاصل بر روی طیف های جداگانه و یا ترکیبی.

6.7 محاسبه ماکزیمم پاسخ سازه ای از طیف پاسخ:

تمامی تلاش های لازم در ترسیم طیف پاسخ انجام شده است و با بدست آوردن پارامترهای طیفی حل مسئله بسیار راحت است.

مثلاً با در دست داشتن طیف شبه شتاب

$$u_o = D = \frac{A}{(\omega_n)^2} = \left(\frac{T_n}{2\pi}\right)^2 A$$

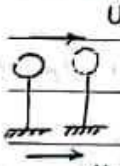
$$f_{so} = kD = mA$$

مثال 6.2 حل شود: $V_{bo} = kD = mA \qquad M_{bo} = hV_{bo}$

6.8 مشخصه های طیف پاسخ:

1) سازه سیستم با سختی بسیار زیاد: تغییرمکان D به علت سختی بسیار بالا تقریباً تمایل صفر شدن است. $\quad T_n \ll T_a: \quad \ddot{u}^t_o = A$

حرکت کل سیستم با حرکت زمین تقریباً برابر است.



$u^t(t) = u_g(t) + u(t)$

$u^t(t) = u_g(t)$

$\ddot{u}^t(t) = \ddot{u}_{go}(t) \qquad \ddot{u}^t_o = \ddot{u}_{go}$

$\ddot{u}^t(t) = -A(t) \Rightarrow A = \ddot{u}_{go} = \ddot{u}^t_o$

linear scales are replotted in Fig. 6.6.3 on logarithmic scales. For a given natural period T_n, the D and A values can be read from the diagonal scales. As an example, for $T_n = 2$ sec, Fig. 6.6.3 gives $D = 7.47$ in. and $A = 0.191$g. (Actually, these numbers cannot be read so accurately from the graph; in this case they were available from Fig. 6.6.2.) The four-way plot is a compact presentation of the three—deformation, pseudo-velocity, and pseudo-acceleration—response spectra, for a single plot of this form replaces the three plots of Fig. 6.6.2.

A response spectrum should cover a wide range of natural vibration periods and several damping values so that it provides the peak response of all possible structures. The period range in Fig. 6.6.3 should be extended because tall buildings and long-span bridges, among other structures, may have longer vibration periods (Fig. 2.1.2), and several damping values should be included to cover the practical range of $\zeta = 0$ to 20%. Figure 6.6.4 shows spectrum curves for $\zeta = 0$, 2, 5, 10, and 20% over the period range 0.02 to 50 sec. This, then, is the response spectrum for the north–south component of ground motion recorded at one location during the Imperial Valley earthquake of May 18, 1940. Because the lateral force or base shear for an SDF system is related through Eq. (6.6.5) to $A/$g, we also plot this normalized pseudo-acceleration spectrum in Fig. 6.6.5. Similarly, because the peak deformation is given by D, we also plot this deformation response spectrum in Fig. 6.6.6.

The response spectrum has proven so useful in earthquake engineering that spectra for virtually all ground motions strong enough to be of engineering interest are now

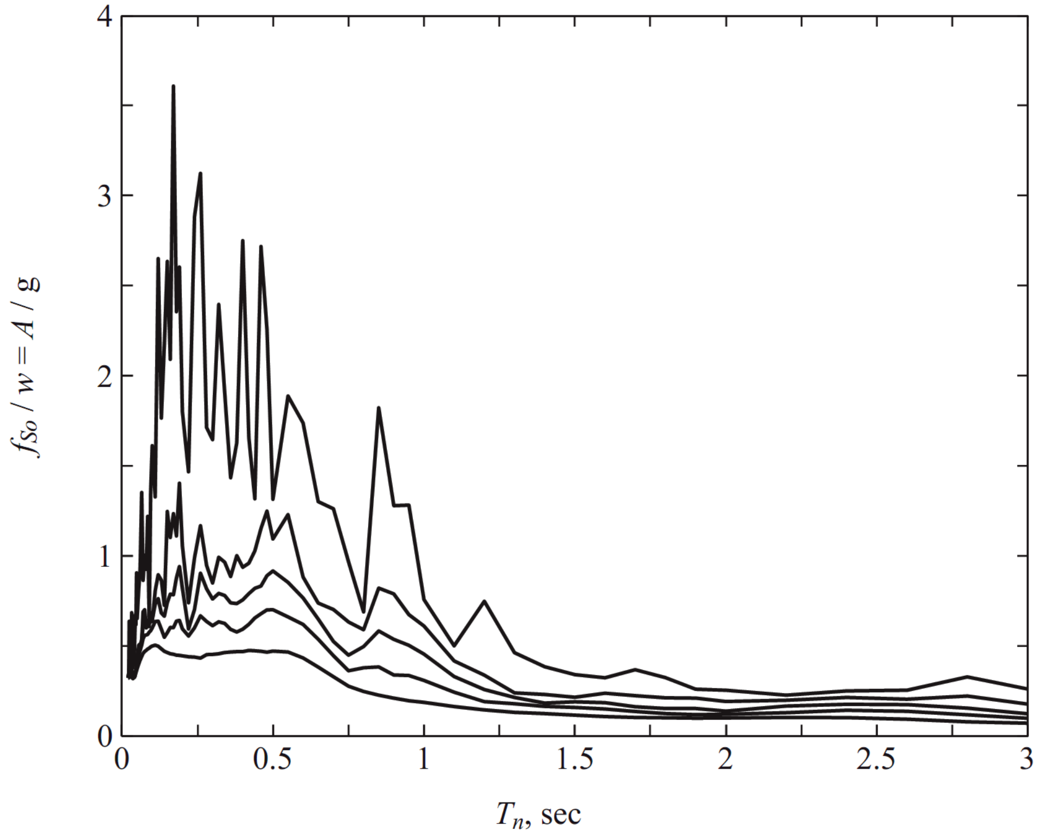


Figure 6.6.5 Normalized pseudo-acceleration, or base shear coefficient, response spectrum for El Centro ground motion; $\zeta = 0$, 2, 5, 10, and 20%.

6.6.5 Construction of Response Spectrum

The response spectrum for a given ground motion component $\ddot{u}_g(t)$ can be developed by implementation of the following steps:

1. Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2. Select the natural vibration period T_n and damping ratio ζ of an SDF system.
3. Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods described in Chapter 5. [In obtaining the

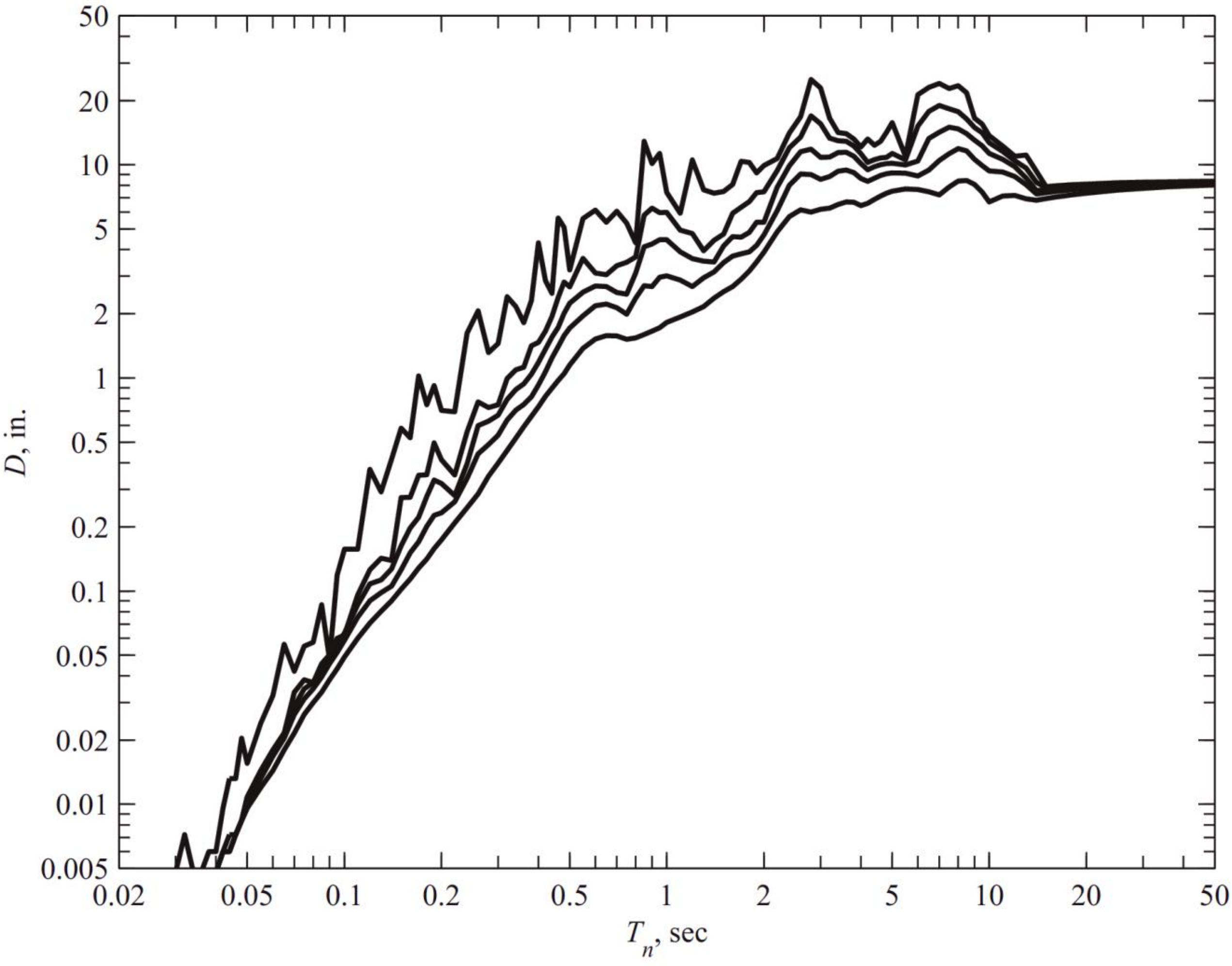


Figure 6.6.6 Deformation response spectrum for El Centro ground motion; $\zeta = 0, 2, 5, 10$, and 20%.

6.6. ساختن طیف پاسخ: فرض کنیم شتاب حرکت زمین $\ddot{u}_g(t)$ به صورت عددی مشخص شده باشد. در این صورت، گام های زیر برای ساختن طیف طی می شوند:

1. عددی ساختن $\ddot{u}_g(t)$ در گام زمانی مشخص، مثلاً $\Delta t = 0.02\,s$
2. انتخاب نسبت میرایی ζ و پریود طبیعی T_n برای یک سیستم یک درجه آزادی (SDF)
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7. نمایش گام ها و نتایج حاصله بر روی طیف های جداگانه و یا ترکیبی.

6.7 محاسبه ماکزیمم پاسخ سازه‌ای از طیف پاسخ:

تمامی تلاش های لازم در توسعه طیف پاسخ انجام شده است و با بدست آوردن پارامترهای طیفی حل مسئله بسیار راحت است.

مثلاً با در دست داشتن طیف شبه شتاب

$$u_o = D = \frac{A}{(\omega_n)^2} = \left(\frac{T_n}{2\pi}\right)^2 A$$

$$f_{s0} = kD = mA$$

مثال 6.2 حل شود:

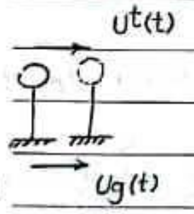
$$V_{b0} = kD = mA \qquad M_{b0} = hV_{b0}$$

6.8 مشخصه های طیف پاسخ:

1) رفتار سیستم با سختی بسیار زیاد: تغییر مکان D به علت سختی بسیار بالا تقریباً قابل صرف نظر است.

حرکت کل سیستم با حرکت زمین تقریباً برابر است.

$T_n \ll T_a$: $\quad \ddot{u}_o^t = A$



$$u^t(t) = u_g(t) + u(t)$$
$$u^t(t) = u_g(t)$$
$$\ddot{u}^t(t) = \ddot{u}_{g0}(t) \qquad \ddot{u}_o^t = \ddot{u}_{g0}$$
$$\ddot{u}^t(t) = -A(t) \Rightarrow A = \ddot{u}_{g0} = \ddot{u}_o^t$$

2. برای یک سیستم بسیار نرم با پریود بسیار بالا $T_n \geqslant T_F$ $D = u_{g0}$

مقدار شتاب کل بسیار کم است و جابجایی با دامنه جابجایی زمین برابر می‌باشد.

$u^t(t) = 0 \qquad \ddot{u}^t(t) = 0$

$u^t(t) = u_g(t) + u(t) = 0$

$u(t) \cong -u_g(t) \Rightarrow D = u_{g0}$

$A(t) = u(t)\,\omega_n^2 = u(t) \times \left(\frac{2\pi}{T_n}\right)^2 \cong 0$ (بسیار بزرگ)

در حالت 1 چرا $\ddot{u}^t(t) = -A(t)$ برقرار است؟ در حالت مطلب:

$m\ddot{u} + ku = -m\ddot{u}_{g0} \sin\omega t$

حل خصوصی $u = A_1 \sin\omega t$ $\quad -m\omega^2 A_1 + kA_1 = -m\ddot{u}_{g0} \quad A_1(-m\omega^2 + m\omega_n^2) = -m\ddot{u}_{g0}$

$\Rightarrow A_1 = \frac{-\ddot{u}_{g0}}{\omega_n^2 - \omega^2} = \frac{\ddot{u}_{g0}}{\omega^2 - \omega_n^2} \Rightarrow u(t) = \frac{\ddot{u}_{g0}}{\omega^2 - \omega_n^2} \sin\omega t$

شبه شتاب $A(t) = u(t)\,\omega_n^2 = \frac{\ddot{u}_{g0}\,\omega_n^2}{\omega^2 - \omega_n^2} \sin\omega t = \frac{\ddot{u}_{g0} \sin\omega t}{\omega/\omega_n^2 - 1}$

$-A(t) = \frac{1}{1 - (\omega/\omega_n)^2} \underbrace{\ddot{u}_{g0} \sin\omega t}_{\ddot{u}_g(t)} \Rightarrow -A(t) = \frac{1}{1 - (\omega/\omega_n)^2} \ddot{u}_g(t)$

هرگاه $\omega_n \gg \omega \Rightarrow -A(t) \cong \ddot{u}_g(t)$

از طرفی با توجه به شکل صفحه قبل $u^t(t) = \underbrace{u(t)}_{0} + u_g(t) \qquad u^t(t) \cong u_g(t)$

$\ddot{u}^t(t) \cong \ddot{u}_g(t) \cong -A(t)$

3. برای یک سیستم با پریود کوتاه $T_a \leqslant T_n \leqslant T_c$ $\quad A = \alpha_A \ddot{u}_{g0} \qquad \alpha_A = \alpha_A(T_n, \xi) > 1$

4. برای یک سیستم با پریود طولانی $T_d \leqslant T_n \leqslant T_F$ $\quad D = \alpha_D u_{g0} \qquad \alpha_D = \alpha_D(T_n, \xi) > 1$

5. برای یک سیستم با پریود میانی $T_c \leqslant T_n \leqslant T_d$ $\quad V = \alpha_V \dot{u}_{g0} \qquad \alpha_V = \alpha_V(\xi) > 1$

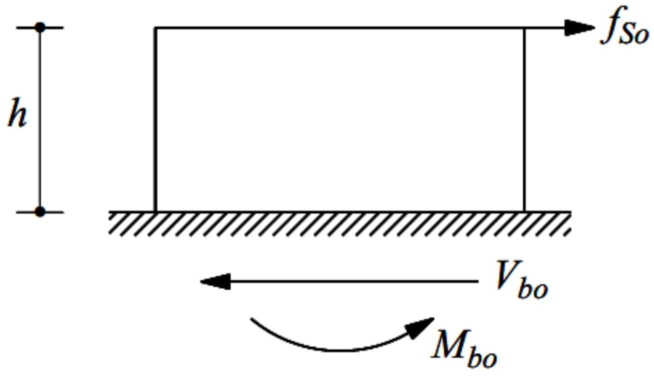


Figure 6.7.1 Peak value of equivalent static force.

by examples. We emphasize again that no further dynamic analysis is required beyond that necessary to determine $u(t)$. In particular, the peak values of shear and overturning moment at the base of the one-story structure are

$$V_{bo} = kD = mA \qquad M_{bo} = hV_{bo} \tag{6.7.3}$$

We note that any one of these response spectra—deformation, pseudo-velocity, or pseudo-acceleration—is sufficient for computing the peak deformations and forces required in structural design. For such applications the velocity or acceleration spectra (defined in Section 6.5) are not required, but for completeness we discuss these spectra briefly at the end of this chapter.

Example 6.2

A 12-ft-long vertical cantilever, a 4-in.-nominal-diameter standard steel pipe, supports a 5200-lb weight attached at the tip as shown in Fig. E6.2. The properties of the pipe are: outside diameter, $d_o = 4.500$ in., inside diameter $d_i = 4.026$ in., thickness $t = 0.237$ in., and second moment of cross-sectional area, $I = 7.23$ in^4, elastic modulus $E = 29{,}000$ ksi, and weight = 10.79 lb/foot length. Determine the peak deformation and bending stress in the cantilever due to the El Centro ground motion. Assume that $\zeta = 2\%$.

Solution The lateral stiffness of this SDF system is

$$k = \frac{3EI}{L^3} = \frac{3(29 \times 10^3)7.23}{(12 \times 12)^3} = 0.211 \text{ kip/in.}$$

The total weight of the pipe is $10.79 \times 12 = 129.5$ lb, which may be neglected relative to the lumped weight of 5200 lb. Thus

$$m = \frac{w}{g} = \frac{5.20}{386} = 0.01347 \text{ kip-sec}^2\text{/in.}$$

The natural vibration frequency and period of the system are

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.211}{0.01347}} = 3.958 \text{ rad/sec} \qquad T_n = 1.59 \text{ sec}$$

From the response spectrum curve for $\zeta = 2\%$ (Fig. E6.2b), for $T_n = 1.59$ sec, $D = 5.0$ in. and $A = 0.20$g. The peak deformation is

$$u_o = D = 5.0 \text{ in.}$$

The peak value of the equivalent static force is

$$f_{So} = \frac{A}{g}w = 0.20 \times 5.2 = 1.04 \text{ kips}$$

The bending moment diagram is shown in Fig. E6.2d with the maximum moment at the base = 12.48 kip-ft. Points A and B shown in Fig. E6.2e are the locations of maximum bending stress:

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(12.48 \times 12)(4.5/2)}{7.23} = 46.5 \text{ ksi}$$

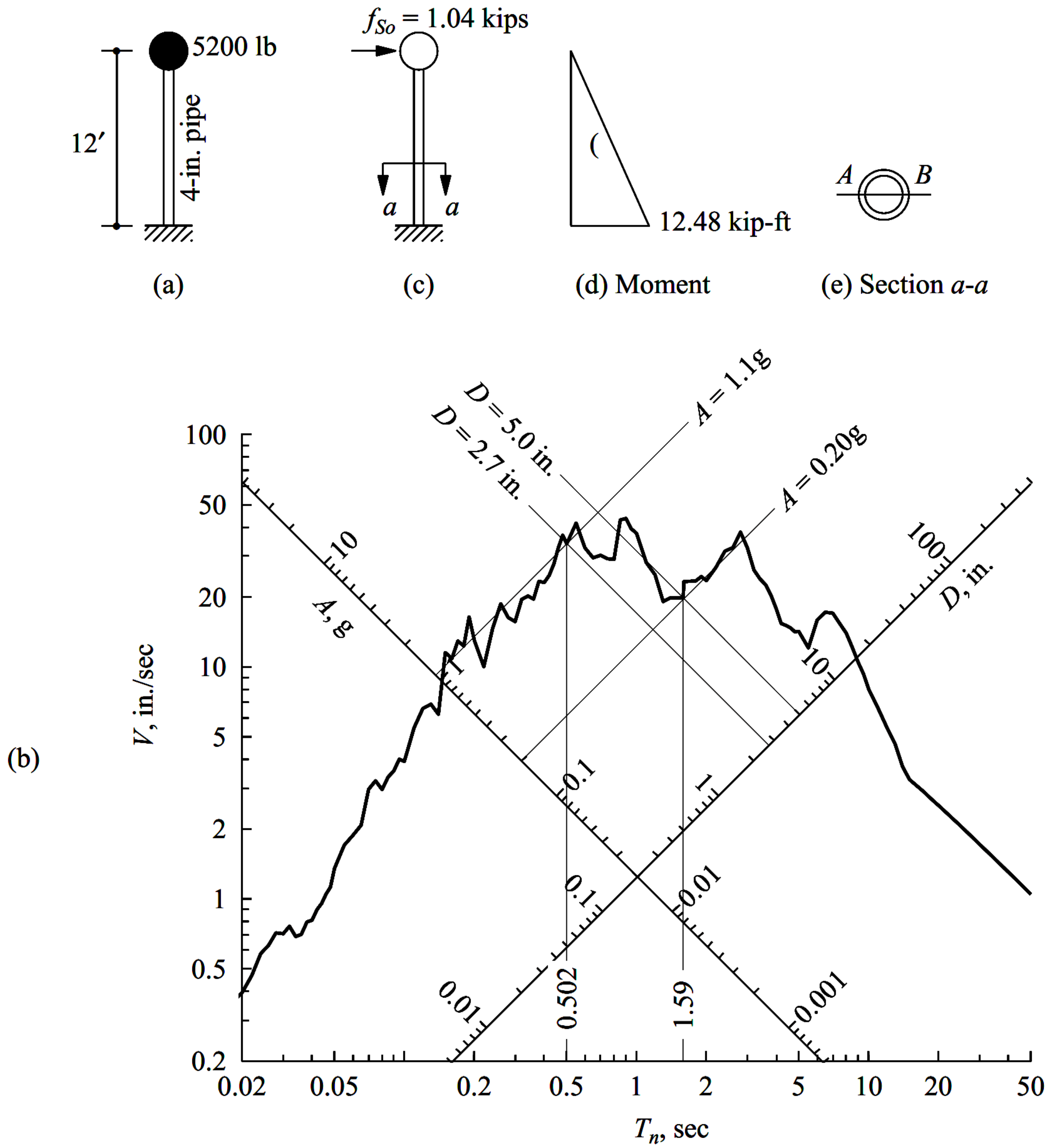


Figure E6.2

Thus, $\sigma = +46.5$ ksi at A and $\sigma = -46.5$ ksi at B, where + denotes tension. The algebraic signs of these stresses are irrelevant because the direction of the peak force is not known, as the pseudo-acceleration spectrum is, by definition, positive.

Example 6.3

The stress computed in Example 6.2 exceeded the allowable stress and the designer decided to increase the size of the pipe to an 8-in.-nominal standard steel pipe. Its properties are $d_o = 8.625$ in., $d_i = 7.981$ in., $t = 0.322$ in., and $I = 72.5$ in^4. Comment on the advantages and disadvantages of using the larger pipe.

Solution

$$k = \frac{3(29 \times 10^3)72.5}{(12 \times 12)^3} = 2.112 \text{ kips/in.}$$

$$\omega_n = \sqrt{\frac{2.112}{0.01347}} = 12.52 \text{ rad/sec} \qquad T_n = 0.502 \text{ sec}$$

From the response spectrum (Fig. E6.2b): $D = 2.7$ in. and $A = 1.1g$. Therefore,

$$u_o = D = 2.7 \text{ in.}$$

$$f_{So} = 1.1 \times 5.2 = 5.72 \text{ kips}$$

$$M_{\text{base}} = 5.72 \times 12 = 68.64 \text{ kip-ft}$$

$$\sigma_{\max} = \frac{(68.64 \times 12)(8.625/2)}{72.5} = 49.0 \text{ ksi}$$

Using the 8-in.-diameter pipe decreases the deformation from 5.0 in. to 2.7 in. However, contrary to the designer's objective, the bending stress increases slightly.

This example points out an important difference between the response of structures to earthquake excitation and to a fixed value of static force. In the latter case, the stress would decrease, obviously, by increasing the member size. In the case of earthquake excitation, the increase in pipe diameter shortens the natural vibration period from 1.59 sec to 0.50 sec, which for this response spectrum has the effect of increasing the equivalent static force f_{So}. Whether the bending stress decreases or increases by increasing the pipe diameter depends on the increase in section modulus, I/c, and the increase or decrease in f_{So}, depending on the response spectrum.

Example 6.4

A small one-story reinforced-concrete building is idealized for purposes of structural analysis as a massless frame supporting a total dead load of 10 kips at the beam level (Fig. E6.4a). The frame is 24 ft wide and 12 ft high. Each column and the beam has a 10-in.-square cross section. Assume that the Young's modulus of concrete is 3×10^3 ksi and the damping ratio for the building is estimated as 5%. Determine the peak response of this frame to the El Centro ground motion. In particular, determine the peak lateral deformation at the beam level and plot the diagram of bending moments at the instant of peak response.

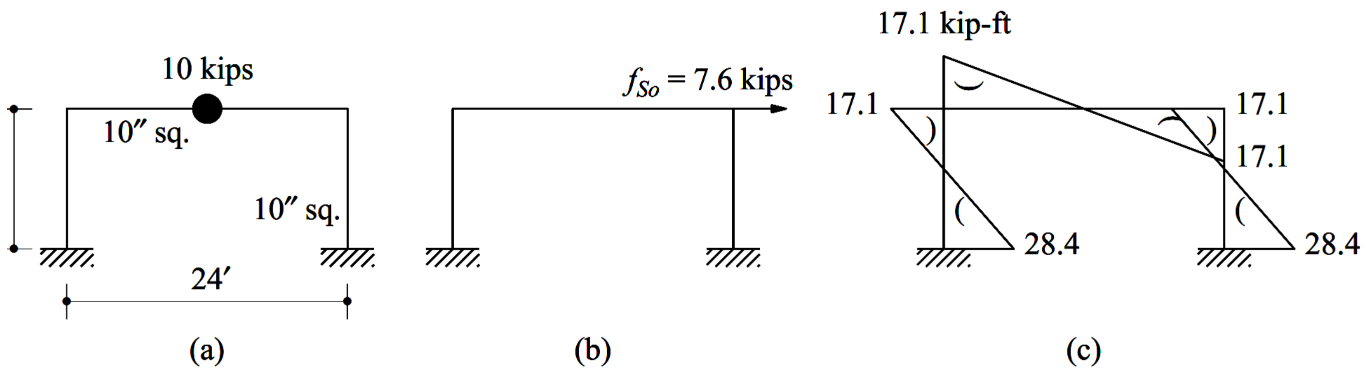


Figure E6.4 (a) Frame; (b) equivalent static force; (c) bending moment diagram.

Solution The lateral stiffness of such a frame was calculated in Chapter 1: $k = 96EI/7h^3$, where EI is the flexural rigidity of the beam and columns and h is the height of the frame. For this particular frame,

$$k = \frac{96(3 \times 10^3)(10^4/12)}{7(12 \times 12)^3} = 11.48 \text{ kips/in.}$$

The natural vibration period is

$$T_n = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{10/386}{11.48}} = 0.30 \text{ sec}$$

For $T_n = 0.3$ and $\zeta = 0.05$, we read from the response spectrum of Fig. 6.6.4: $D = 0.67$ in. and $A = 0.76$g. Peak deformation: $u_o = D = 0.67$ in. Equivalent static force: $f_{So} = (A/g)w = 0.76 \times 10 = 7.6$ kips. Static analysis of the frame for this lateral force, shown in Fig. E6.4b, gives the bending moments that are plotted in Fig. E6.4c.

Example 6.5

The frame of Example 6.4 is modified for use in a building to be located on sloping ground (Fig. E6.5). The beam is now made much stiffer than the columns and can be assumed to be rigid. The cross sections of the two columns are 10 in. square, as before, but their lengths are 12 ft and 24 ft, respectively. Determine the base shears in the two columns at the instant of peak response due to the El Centro ground motion. Assume the damping ratio to be 5%.

Solution

1. *Compute the natural vibration period.*

$$k = \frac{12(3 \times 10^3)(10^4/12)}{(12 \times 12)^3} + \frac{12(3 \times 10^3)(10^4/12)}{(24 \times 12)^3}$$

$$= 10.05 + 1.26 = 11.31 \text{ kips/in.}$$

$$T_n = 2\pi\sqrt{\frac{10/386}{11.31}} = 0.30 \text{ sec}$$

2. *Compute the shear force at the base of the short and long columns.*

$$u_o = D = 0.67 \text{ in.}, \qquad A = 0.76\text{g}$$

$$V_{\text{short}} = k_{\text{short}} u_o = (10.05)0.67 = 6.73 \text{ kips}$$

$$V_{\text{long}} = k_{\text{long}} u_o = (1.26)0.67 = 0.84 \text{ kip}$$

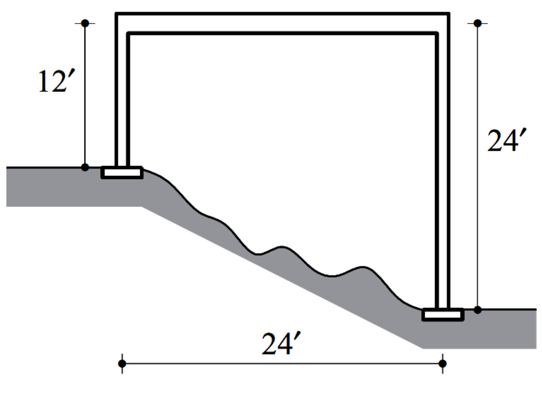


Figure E6.5

Observe that both columns go through equal deformation. Undergoing equal deformations, the stiffer column carries a greater force than the flexible column; the lateral force is distributed to the elements in proportion to their relative stiffnesses. Sometimes this basic principle has, inadvertently, not been recognized in building design, leading to unanticipated damage of the stiffer elements.

Example 6.6

For the three-span box-girder bridge of Example 1.3, determine the base shear in each of the six columns of the two bents due to El Centro ground motion applied in the longitudinal direction. Assume the damping ratio to be 5%.

Solution The weight of the bridge deck was computed in Example 1.3: $w = 6919$ kips. The natural period of longitudinal vibration of the bridge was computed in Example 2.2: $T_n = 0.573$ sec. For $T_n = 0.573$ sec and $\zeta = 0.05$, we read from the response spectrum of Fig. 6.6.4: $D = 2.591$ in. and $A = 0.807\text{g}$.

All the columns have the same stiffness and they go through equal deformation $u_o = D = 2.591$ in. Thus, the base shear will be the same in all columns, which can be computed in one of two ways: The total equivalent static force on the bridge is [from Eq. (6.6.5)]

$$f_{so} = 0.807 \times 6919 = 5584 \text{ kips}$$

Base shear for one column, $V_b = 5584 \div 6 = 931$ kips. Alternatively, the base shear in each column is

$$V_b = k_{\text{col}} u_o = 4313 \times \frac{2.591}{12} = 931 \text{ kips}$$

6.8 RESPONSE SPECTRUM CHARACTERISTICS

We now study the important properties of earthquake response spectra. Figure 6.8.1 shows the response spectrum for El Centro ground motion together with $\ddot{u}_{go}$, $\dot{u}_{go}$, and u_{go}, the peak values of ground acceleration, ground velocity, and ground displacement, respectively, identified in Fig. 6.1.4. To show more directly the relationship between the response spectrum and the ground motion parameters, the data of Fig. 6.8.1 have been presented again in Fig. 6.8.2 using normalized scales: D/u_{go}, $V/\dot{u}_{go}$, and $A/\ddot{u}_{go}$. Figure 6.8.3 shows one of the spectrum curves of Fig. 6.8.2, the one for 5% damping, together with an idealized version shown in dashed lines; the latter will provide a basis for constructing smooth design spectra directly from the peak ground motion parameters (see Section 6.9). Based on Figs. 6.8.1 to 6.8.3, we first study the properties of the response spectrum over various ranges of the natural vibration period of the system separated by the period values at a, b, c, d, e, and f: $T_a = 0.035$ sec, $T_b = 0.125$, $T_c = 0.5$, $T_d = 3.0$, $T_e = 10$, and $T_f = 15$ sec. Subsequently, we identify the effects of damping on spectrum ordinates.

For systems with very short period, say $T_n < T_a = 0.035$ sec, the peak pseudo-acceleration A approaches $\ddot{u}_{go}$ and D is very small. This trend can be understood based on physical reasoning. For a fixed mass, a very short-period system is extremely stiff or essentially rigid. Such a system would be expected to undergo very little deformation and its mass would move rigidly with the ground; its peak acceleration should be approximately

تقسیم‌بندی نواحی طیف:

۱- ناحیه حساس به شتاب: $T_n < T_c$ ؛ پاسخ سازه مستقیماً متناسب با $\ddot{u}_{g0}$ است.

2- ناحیه حساس به سرعت: $T_c \leq T_n \leq T_d$ ؛ پاسخ سازه مستقیماً متناسب $\dot{u}_{g0}$ است.

3- ناحیه حساس به جابجایی، $T_n > T_d$: جابجایی D متناسب با u_{g0} است.

ایده‌آل نمودن نمودار طیف: ایده‌آل نمودن طیف و محاسبه پریودهای T_f و T_a دارای یک جواب دقیق منطقاً نیست اما به هر حال با استفاده از روش‌های متداول نیست کردن نمودار می‌توان آنرا بدست آورد.

تفاوت در نمودارهای طیف: پریودهای T_f و T_a و ضرایب افزایش آنها $(\alpha_A, \alpha_V, \alpha_D)$ برای حرکات متفاوت زمین یکسان نیستند بلکه ماهیت ذاتی احتمالاتی آنها بر این پارامترها تأثیرگذار است. به علاوه سایر شرایط مانند بزرگای زلزله، نزدیک‌ترین فاصله تا گسل، مکانیسم گسل و جنس خاک ساختگاه بر این پارامترها تأثیرگذار است. در هر حال، شکل طیف چهار جانبه دارای نواحی سه‌گانه حساس به شتاب، سرعت و جابجایی خواهد بود.

پاسخ (درست است بگوییم بر A بی‌تأثیر است)

تأثیر میرایی بر طیف پاسخ: ۱. برای شرایط $T_n < T_a$ وقتی $T_n \to 0$ میرایی بر سیستم صلب بی تأثیر است.

2. برای شرایط $T_n > T_f$ وقتی $T_n \to \infty$ میرایی بر پاسخ سیستم بسیار نرم بی تأثیر است. [illegible]

3. در سایر نواحی میرایی در کاهش پاسخ موثر است و بیشترین تأثیر در ناحیه حساس به سرعت است.

تأثیر متقابل میرایی و حرکت زمین:

۱. چنانچه حرکت زمین مجموعه‌ای از چرخه‌های هارمونیک باشد (مانند زلزله مکزیکوسیتی) تأثیر میرایی بسیار مهم است به ویژه در شرایط نزدیک به تشدید

2. چنانچه حرکت زمین مانند یک پالس باشد تأثیر میرایی اندک است، مانند حرکات نزدیک به گسل

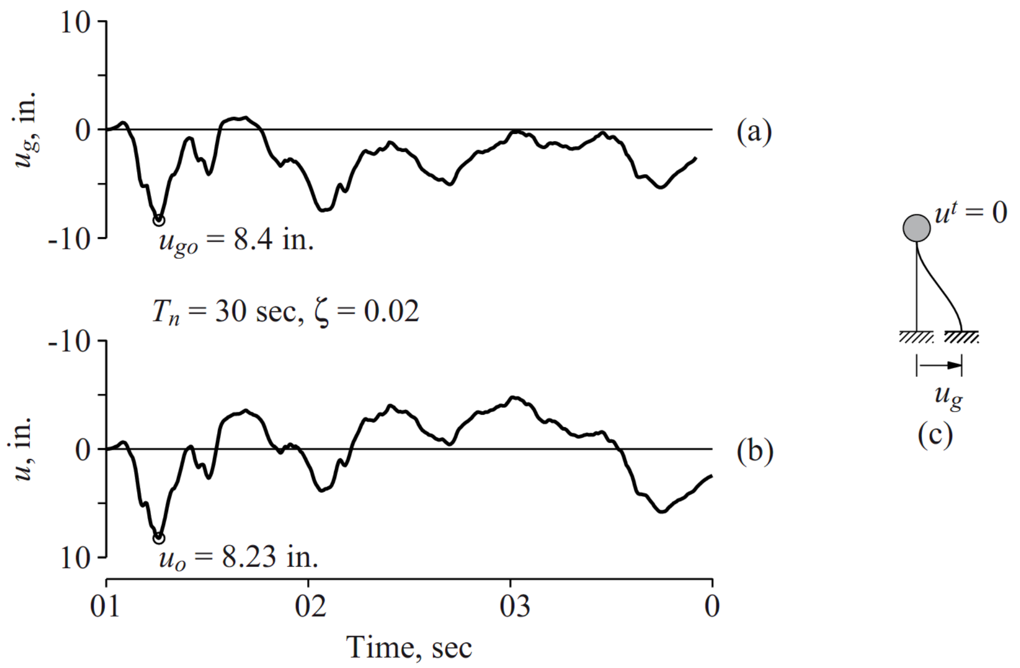


Figure 6.8.5 (a) El Centro ground displacement; (b) deformation response of SDF system with $T_n = 30$ sec and $\zeta = 2\%$; (c) very flexible system.

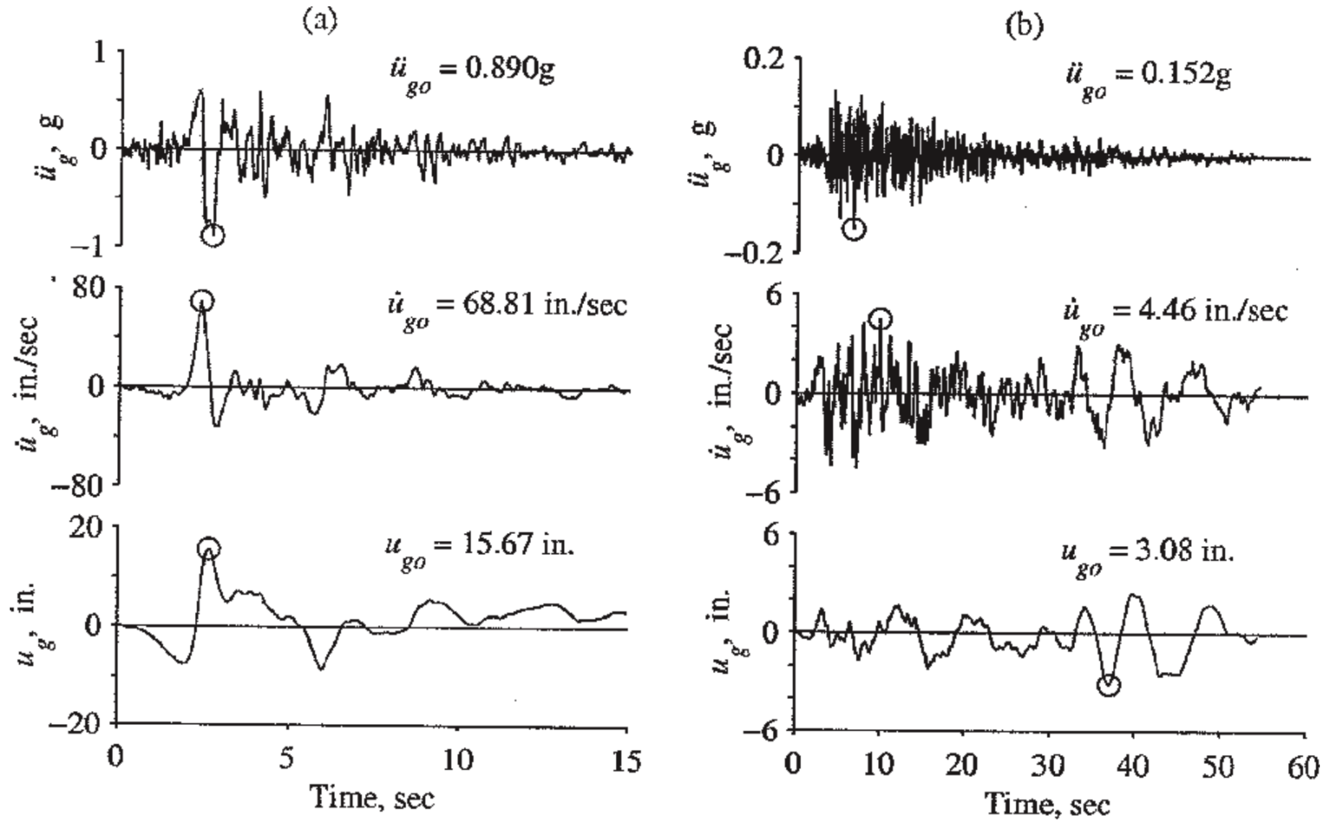


Figure 6.8.6 Fault-normal component of ground motions recorded at (a) Rinaldi Receiving Station, 1994 Northridge earthquake, and (b) Taft, 1952 Kern County earthquake.

تأثیر میرایی در کاهش پاسخ سازه از ٪2 ← ٪5 = ζ به مراتب بیشتر از ٪12 ← ٪10 = ζ است.

با سازوکارگی میراگر در سازه‌های بلند به خوبی می‌توان پاسخ سازه در هنگام باد یا زلزله را کاهش داد. مثلاً 10,000 میراگر در ارتفاع برج بلند World Trade Center در نیویورک به کار رفته است.

مفهوم طیف طرح الاستیک:

هدف از توسعه طیف طرح: طراحی سازه‌های جدید

ارزیابی ایمنی و عملکرد لرزه‌ای سازه‌های موجود

با توجه به این که ۱) طیف پاسخ هر زلزله متفاوت با زلزله‌ی دیگر است و ۲) ماهیت زلزله‌ای که در آینده رخ خواهد رخ می‌دهد، مشخص نیست؛ بنابراین طیف طرح باید طیف هموار شده‌ای باشد که توسط مجموعه‌ای از خطوط در سطوح متفاوت برای ارائه می‌گردد. این طیف باید دربرگیرنده مجموعه‌ای از حرکات زمین باشد که در منطقه رخ داده و یا در مناطق مشابه دیگر رخ داده است. فاکتورهای لازم برای تشابه دو ناحیه از نظر لرزه‌ای عبارتند از:

۱. بزرگی زلزله؛ 2. فاصله تا محل گسل 3. شرایط محیط خاک در مسیر انتشار امواج تا ساختگاه 4. شرایط خاک ساختگاه 5. مکانیسم گسل

نحوه ساختن طیف طرح:

طیف طرح براساس آنالیز آماری روی مجموعه‌ای از طیف‌های پاسخ از حرکات زمین بدست می‌آید. بنابراین اولین گام در توسعه طیف طرح، هم پایه‌سازی مجموعه‌ی حرکات زمین مثلاً بر پایه بیشینه شتاب $(\ddot{u}_{g0} = 1g)$ است. پس از هم پایه‌سازی طیف پاسخ حرکت از شتاب‌نگاشت‌ها توسعه داده می‌شود. به این ترتیب برای

هر پریود T_n به تعداد حرکت‌های زمین (I) پارامترهای طیفی D^i، V^i، A^i ($i = 1, 2, \dots, I$) وجود خواهند داشت که عبارتند از:

با در نظر گرفتن پریود T_n و فرض تابع توزیع تجمعی پارامترهای توزیع شامل میانه و انحراف معیار مشخص می‌گردند. به این ترتیب می‌توان

$$\frac{\text{میانه}}{\text{انحراف معیار}} = \frac{\mu}{\sigma} = \text{ضریب تغییرات}$$

طیف را برای میانه نتایج و یا میانه + یک انحراف معیار گسترش داد.

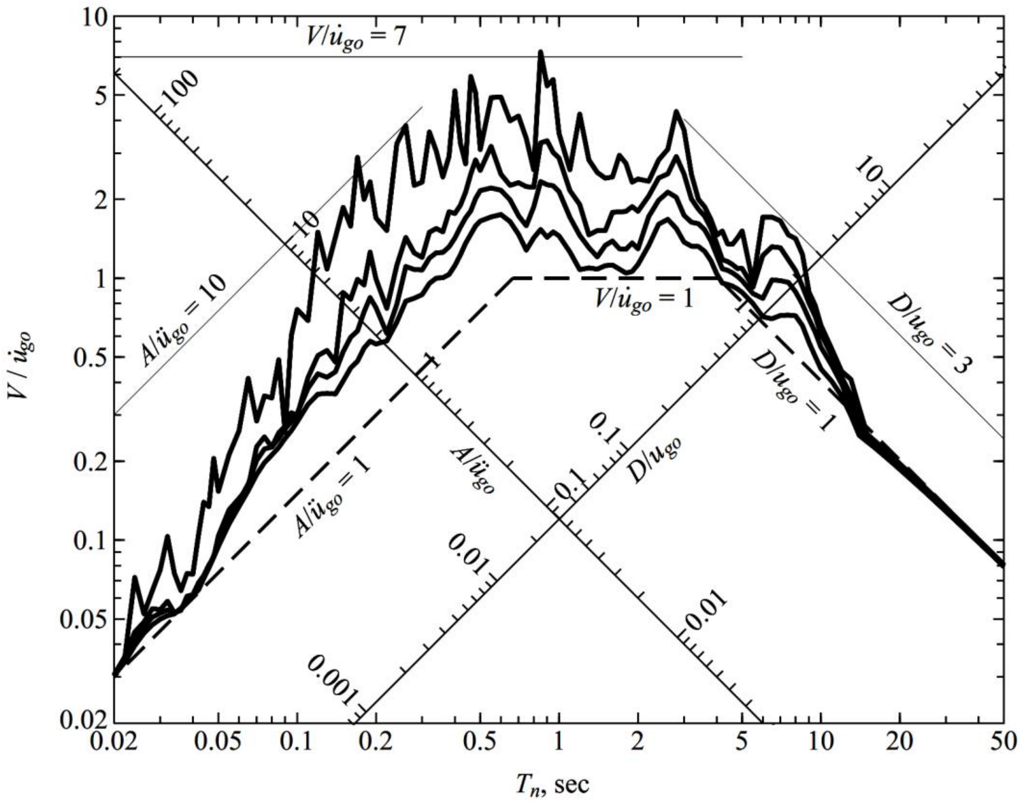


Figure 6.8.2 Response spectrum for El Centro ground motion plotted with normalized scales $A/\ddot{u}_{go}$, $V/\dot{u}_{go}$, and D/u_{go}; $\zeta = 0$, 2, 5, and 10%.

For short-period systems with T_n between $T_a = 0.035$ sec and $T_c = 0.50$ sec, A exceeds $\ddot{u}_{go}$, with the amplification depending on T_n and ζ. Over a portion of this period range, $T_b = 0.125$ sec to $T_c = 0.5$ sec, A may be idealized as constant at a value equal to $\ddot{u}_{go}$ amplified by a factor depending on ζ.

For long-period systems with T_n between $T_d = 3$ sec and $T_f = 15$ sec, D generally exceeds u_{go}, with the amplification depending on T_n and ζ. Over a portion of this period range, $T_d = 3.0$ sec to $T_e = 10$ sec, D may be idealized as constant at a value equal to u_{go} amplified by a factor depending on ζ.

For intermediate-period systems with T_n between $T_c = 0.5$ sec and $T_d = 3.0$ sec, V exceeds $\dot{u}_{go}$. Over this period range, V may be idealized as constant at a value equal to $\dot{u}_{go}$, amplified by a factor depending on ζ.

Based on these observations, it is logical to divide the spectrum into three period ranges (Fig. 6.8.3). The long-period region to the right of point d, $T_n > T_d$, is called the *displacement-sensitive region* because structural response is related most directly to ground displacement. The short-period region to the left of point c, $T_n < T_c$, is called the *acceleration-sensitive region* because structural response is most directly related to ground acceleration. The intermediate period region between points c and d, $T_c < T_n < T_d$, is called the *velocity-sensitive region* because structural response appears to be better related to ground velocity than to other ground motion parameters. For a particular ground motion,

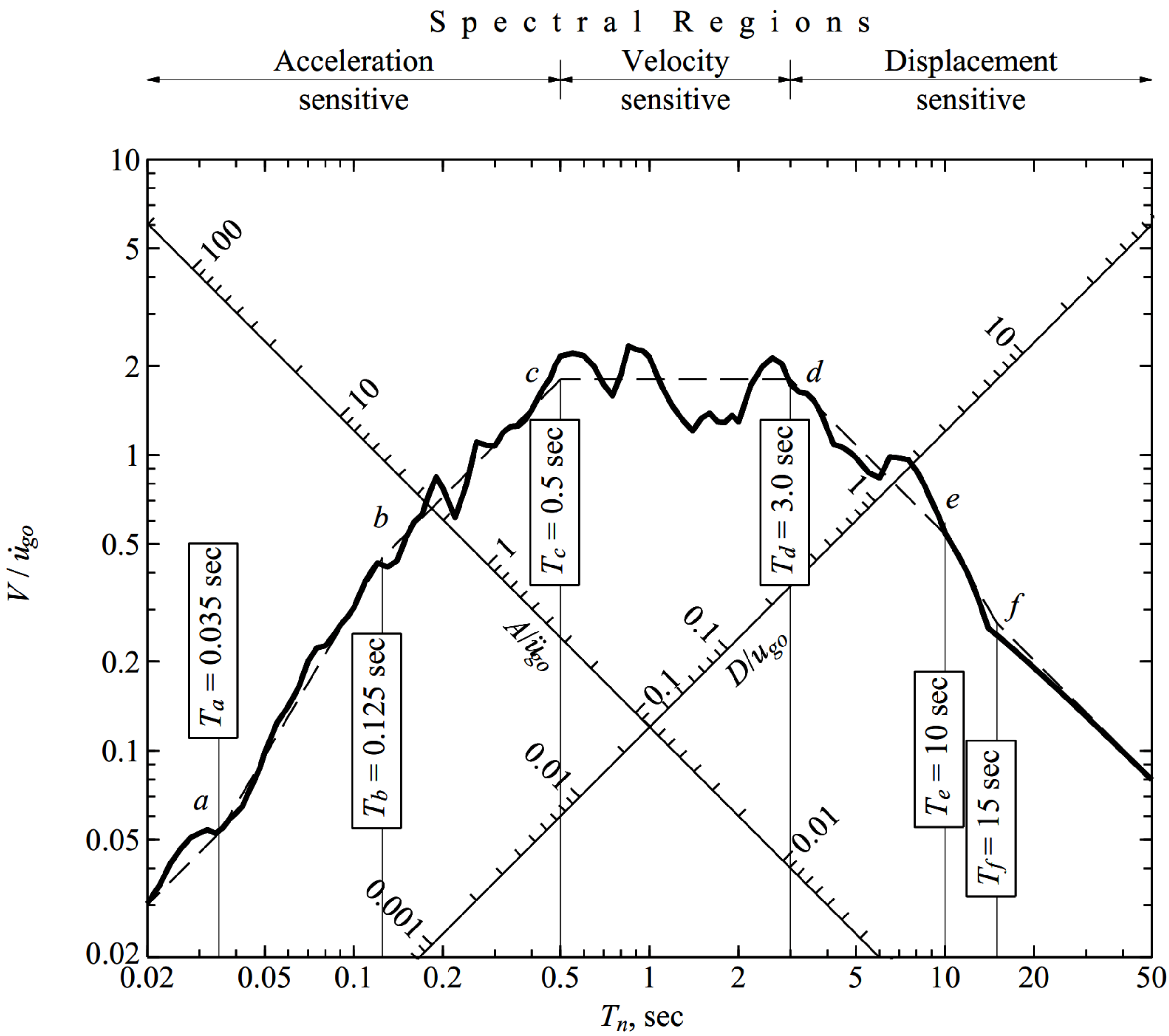


Figure 6.8.3 Response spectrum for El Centro ground motion shown by a solid line together with an idealized version shown by a dashed line; $\zeta = 5\%$.

the periods T_a, T_b, T_e, and T_f on the idealized spectrum are independent of damping, but T_c and T_d vary with damping.

The preceding observations and discussion have brought out the usefulness of the four-way logarithmic plot of the combined deformation, pseudo-velocity, and pseudo-acceleration response spectra. These observations would be difficult to glean from the three individual spectra.

Idealizing the spectrum by a series of straight lines *a*–*b*–*c*–*d*–*e*–*f* in the four-way logarithmic plot is obviously not a precise process. For a given ground motion, the period values associated with the points *a*, *b*, *c*, *d*, *e*, and *f* and the amplification factors for the segments *b*–*c*, *c*–*d*, and *d*–*e* are somewhat judgmental in the way we have approached them. However, formal curve-fitting techniques can be used to replace the actual spectrum by an idealized spectrum of a selected shape. In any case, the idealized spectrum in Fig. 6.8.3 is not a close approximation to the actual spectrum. This may not be visually apparent but becomes obvious when we note that the scales are logarithmic. As we shall see in the next section, the greatest benefit of the idealized spectrum is in constructing a design spectrum representative of many ground motions.

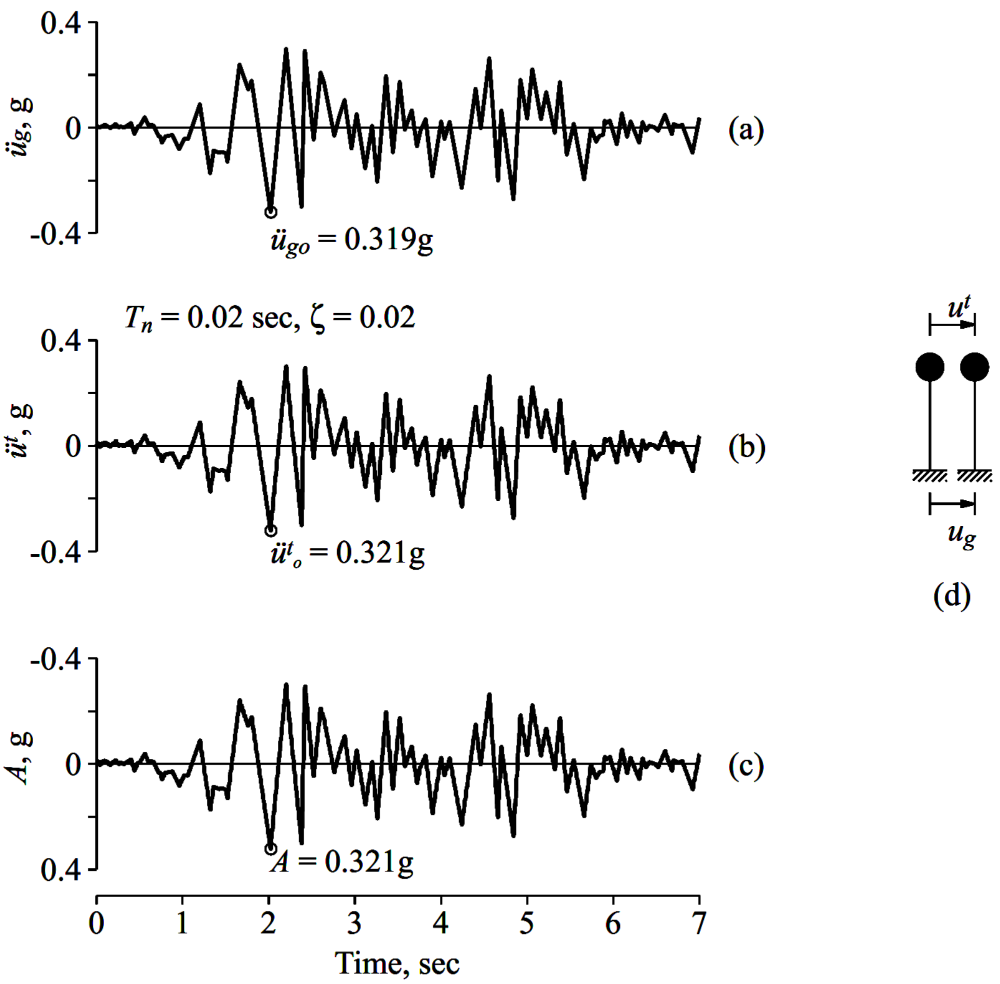


Figure 6.8.4 (a) El Centro ground acceleration; (b) total acceleration response of an SDF system with $T_n = 0.02$ sec and $\zeta = 2\%$; (c) pseudo-acceleration response of the same system; (d) rigid system.

The periods T_a, T_b, T_c, T_d, T_e, and T_f separating spectral regions and the amplification factors for the segments b–c, c–d, and d–e depend on the time variation of ground motion, in particular, the relative values of peak ground acceleration, velocity, and displacement, as indicated by their ratios: $\dot{u}_{go}/\ddot{u}_{go}$ and $u_{go}/\ddot{u}_{go}$. These ground motion characteristics depend on the earthquake magnitude, fault-to-site distance, source-to-site geology, and soil conditions at the site.

Ground motions recorded within the near-fault region of an earthquake at stations located toward the direction of the fault rupture are qualitatively quite different from the usual far-fault earthquake ground motions. The fault-normal component of a ground motion recorded in the near-fault region of the Northridge, California, earthquake of January 17, 1994 displays a long-period pulse in the acceleration history that appears as a coherent pulse in the velocity and displacements histories (Fig. 6.8.6a). Such a pronounced pulse does not exist in ground motions recorded at locations away from the near-fault region, such as the Taft record obtained from the Kern County, California, earthquake of July 21, 1952 (Fig. 6.8.6b).

The ratios $\dot{u}_{go}/\ddot{u}_{go}$ and $u_{go}/\ddot{u}_{go}$ are very different between the fault normal components of near- and far-fault motions. As apparent from the peak values noted in Fig. 6.8.6,

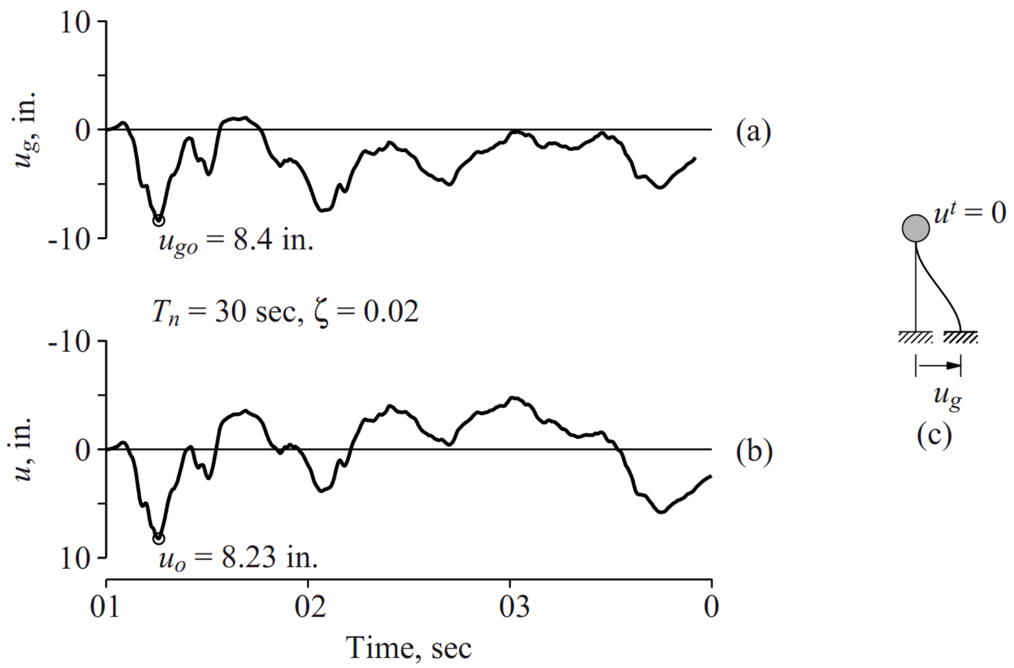


Figure 6.8.5 (a) El Centro ground displacement; (b) deformation response of SDF system with $T_n = 30$ sec and $\zeta = 2\%$; (c) very flexible system.

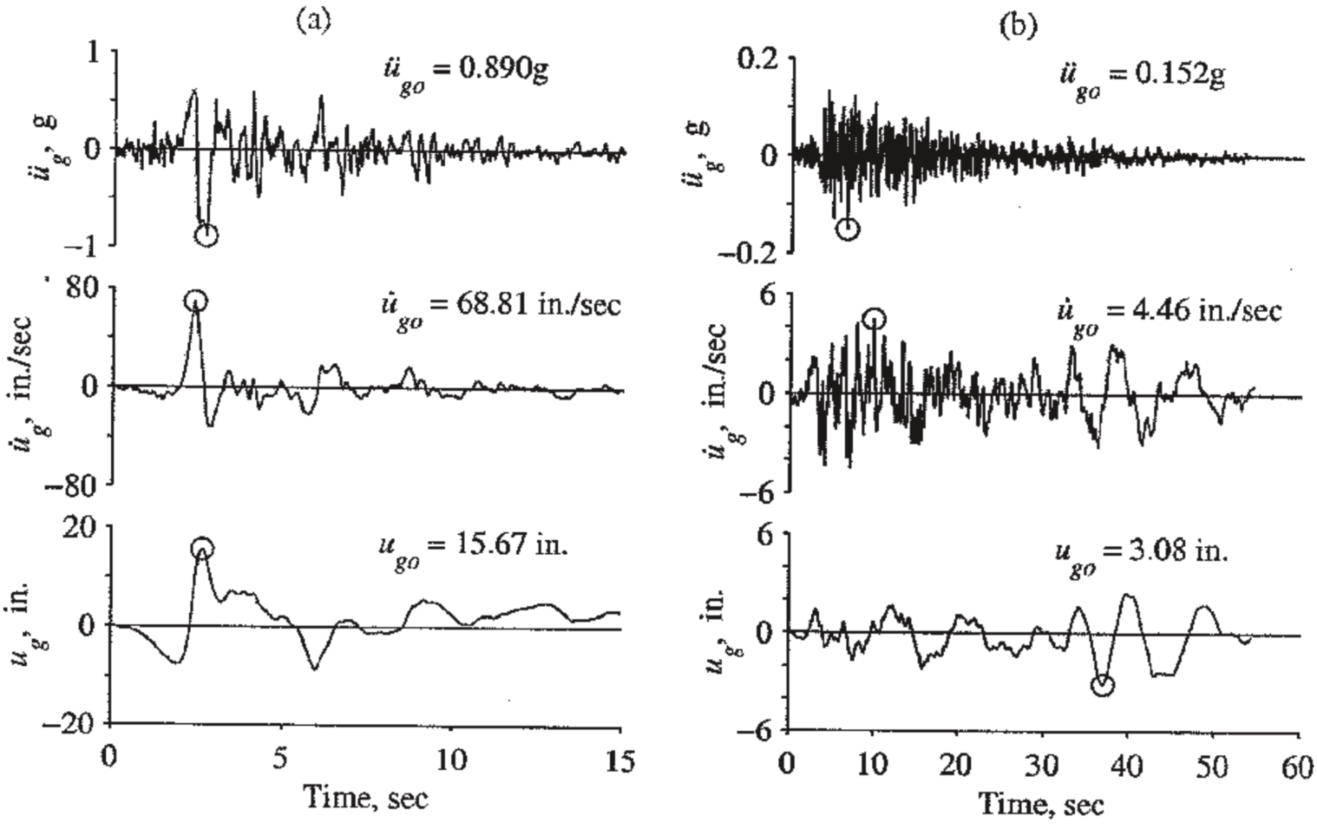


Figure 6.8.6 Fault-normal component of ground motions recorded at (a) Rinaldi Receiving Station, 1994 Northridge earthquake, and (b) Taft, 1952 Kern County earthquake.